بسم الله الرحمن الرحيم

King Abdulaziz University

Exam One

Dr.Hamed Al-Sulami

- أنقر على Start لبداء الاختبار.
- يحتوي هذا الأختبار على خمسة وأربعين سؤالاً.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء الله.

Calculus I
Math110

May 4, 2011

hhaalsalmi@kau.edu.sa
Version 1.0

Enter Name:

I.D. Number:

Start Answer each of the following.

1. The domain of $f(x) = \dfrac{x}{1 - e^x}$ is

- $(-\infty, 0) \cup (0, \infty)$
- $(-\infty, 0]$
- $\mathbb{R}$
- $[0, \infty)$

2. The domain of $f(x) = \sqrt{2^x - 32}$

- $(-\infty, 5) \cup (5, \infty)$
- $\mathbb{R}$
- $(5, \infty)$
- $[5, \infty)$

3. $\lim_{t \to 0} \tan\left(4\pi - \frac{\pi \sin t}{t}\right) =$

- ☐ 1
- ☐ 3π
- ☐ 0
- ☐ -1

4. $\lim_{x \to \infty} \frac{1 - 3x^2}{4x^2 + x + 1} =$

- ☐ $\frac{3}{4}$
- ☐ $\frac{-4}{3}$
- ☐ 1
- ☐ $\frac{-3}{4}$

5. $\lim_{x\to -3^+} \dfrac{1}{x^2-9} =$

- ☐ 0
- ☐ 1
- ☐ $-\infty$
- ☐ ∞

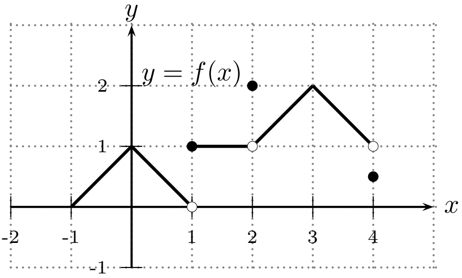
6. $\lim_{x\to 4} \dfrac{x-4}{x^2-2x-8} =$

- ☐ $\dfrac{-1}{6}$
- ☐ $\dfrac{1}{2}$
- ☐ 6
- ☐ $\dfrac{1}{6}$

7. $\lim_{x \to 1} \dfrac{x-1}{\sqrt{x^2+3}-2} =$

- ☐ 2
- ☐ 0
- ☐ -2
- ☐ 1

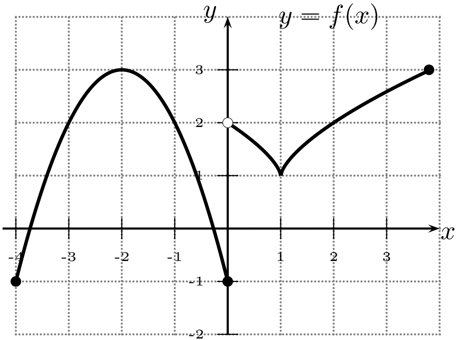
8. The accompanying figure shows the graph of $y = f(x)$. Then $f(x)$ is continuous at $x = 0$.



- ☐ True
- ☐ False

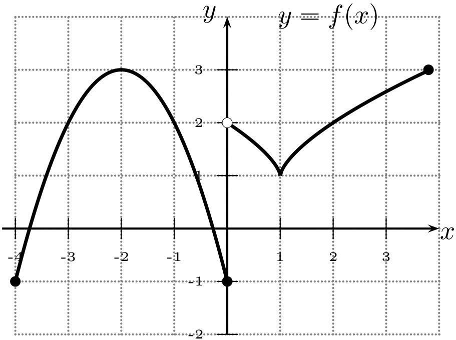
9. The accompanying figure shows the graph of $y = f(x)$. Then $f'(-2) =$.

- [ ] -3
- [ ] 0
- [ ] 1
- [ ] 3



10. The accompanying figure shows the graph of $y = f(x)$. Then f is differentiable at 1.

☐ True ☐ False



11. If $y = \sqrt{2}$, then $y' =$

- ☐ $\dfrac{1}{\sqrt{2}}$
- ☐ 0
- ☐ $\sqrt{2}$
- ☐ $\dfrac{1}{2\sqrt{2}}$

12. $\displaystyle\lim_{x \to -1} \frac{|x-1|-2}{x+1} =$

- ☐ 0
- ☐ Does Not Exist
- ☐ 1
- ☐ -1

13. $y = \dfrac{3x+1}{x-1}$, then $y' =$

- ☐ $\dfrac{-4}{(x-2)^2}$
- ☐ $\dfrac{-6x-8}{(x-2)^2}$
- ☐ $\dfrac{-8}{(x-2)^2}$
- ☐ $\dfrac{4}{(x-2)^2}$

14. The function $f(x) = \sqrt{2-|x|}$ is continuous on

- ☐ $(-\infty, -2] \cup [2, \infty)$
- ☐ $[-2, 2]$
- ☐ $(-\infty, 2) \cup (2, \infty)$
- ☐ $[2, \infty)$

15. An equation for the tangent line to the curve $f(x) = x^2 + 2x$ at $x = 1$ is

- ☐ $y = -4x + 7$
- ☐ $y = 4x + 1$
- ☐ $y = 4x - 1$
- ☐ $y = -4x - 1$

16. The graph of the function $F(x) = x^3 - 27x$, has a horizontal tangent line at

- ☐ $x = \pm 3$
- ☐ $x = 3$
- ☐ $x = -3$
- ☐ $x = 0$

17. If $\dfrac{x^2-2x-3}{x-3} \le f(x) \le \sqrt{3x+7}, \quad x \in [2,4], x \neq 3$, then $\lim\limits_{x\to 3} f(x) =$

- 1
- 4
- -4
- 0

18. The curve $f(x) = \dfrac{x^2+2x-3}{x^3-9x}$ has a vertical asymptote at

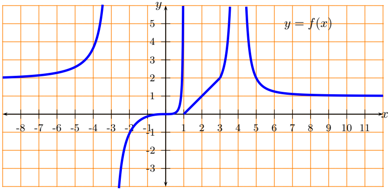
- $x=0, \quad x=3$
- $x=0, \quad x=\pm 3$
- $x=0, \quad x=-3$
- $y=0, \quad y=3$

19. If $y = \dfrac{x^3 + x^2 - x}{x^2}$, then $y' =$

☐ $1 - \dfrac{1}{x^2}$ ☐ $1 - \dfrac{1}{x}$

☐ $1 + \dfrac{1}{x}$ ☐ $1 + \dfrac{1}{x^2}$

20. The accompanying figure shows the graph of $y = f(x)$. Then $\lim_{x\to\infty} f(x) =$



☐ ∞ ☐ 2

☐ 0 ☐ 1

21. $\lim_{x \to -3} \dfrac{x^3 + 27}{x^2 - 9} =$

- ☐ $-\dfrac{3}{2}$
- ☐ $\dfrac{9}{2}$
- ☐ $-\dfrac{9}{2}$
- ☐ 0

22. The horizontal asymptotes for $f(x) = \dfrac{3x - 1}{\sqrt{x^2 + x + 1}}$.

- ☐ $y = \pm 3$
- ☐ $y = 3$
- ☐ $x = \pm 3$
- ☐ $y = -3$

23. $\lim_{x\to -\infty}(x+\sqrt{x^2+x+1}) =$

- ☐ $\frac{1}{2}$
- ☐ ∞
- ☐ $\frac{-1}{2}$
- ☐ $-\infty$

24. The inverse function of $f(x) = \sqrt{x+1}$ is

- ☐ $f^{-1}(x) = x^2 - 1, \quad x \leq 0$
- ☐ $f^{-1}(x) = x^2 - 1, \quad x \geq 0$
- ☐ $f^{-1}(x) = x^2 - 1$
- ☐ $f^{-1}(x) = x^2 - 1, \quad x > 0$

25. The inverse function of $f(x) = \dfrac{e^x}{1+3e^x}$ is

- ☐ $\ln\left(\dfrac{1-3x}{x}\right)$
- ☐ $\dfrac{1-3x}{x}$
- ☐ $\ln\left(\dfrac{x}{1-3x}\right)$
- ☐ $\dfrac{x}{1-3x}$

26. The solution of $\ln(\ln x) = 1$ is

- ☐ 1
- ☐ e^e
- ☐ e
- ☐ e^{-1}

27. The domain of $f(x) = \ln(3 - x)$

- ☐ $(-\infty, 3]$
- ☐ $(-\infty, 3)$
- ☐ $(-3, \infty)$
- ☐ $[-3, \infty)$

28. The domain of $f(x) = \ln(2 + \ln x)$

- ☐ (e^{-2}, ∞)
- ☐ (e^{-1}, ∞)
- ☐ $(0, \infty)$
- ☐ $[e^{-2}, \infty)$

29. $\lim_{x \to e^+} [\![x]\!] =$

- ☐ 0
- ☐ 3
- ☐ 2
- ☐ DNE

30. If $f(x) = \begin{cases} \dfrac{x^3 - 27}{x - 3}, & \text{if } x < 3; \\ 27, & \text{if } x = 3; \\ \dfrac{x^2 + 21x - 72}{x - 3}, & \text{if } x > 3. \end{cases}$, then $\lim_{x \to 3} f(x) =$

- ☐ 3
- ☐ 27
- ☐ -27
- ☐ DNE

31. If x is non-integers real number, then $\frac{d}{dx}(\llbracket x \rrbracket) =$

- ☐ 0
- ☐ x
- ☐ $\llbracket x \rrbracket$
- ☐ DNE

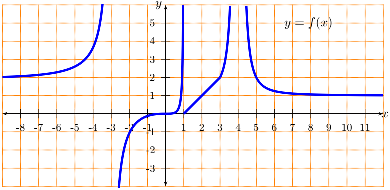
32. $\sin^{-1}\left(\sin\left(\frac{5\pi}{2}\right)\right) =$

- ☐ $\frac{5\pi}{2}$
- ☐ $\frac{-\pi}{2}$
- ☐ 0
- ☐ $\frac{\pi}{2}$

33. $\lim_{x\to 3}\left([\![x]\!]+[\![-x]\!]\right)=$

- [ ] 1
- [ ] DNE
- [ ] 0
- [ ] -1

34. The accompanying figure shows the graph of $y=f(x)$. Then the vertical asymptotes for f are



- [ ] $y=-3,\ y=1,\ y=4$
- [ ] $x=2,\ x=1$
- [ ] $x=-3,\ x=4$
- [ ] $x=-3,\ x=1,\ x=4$

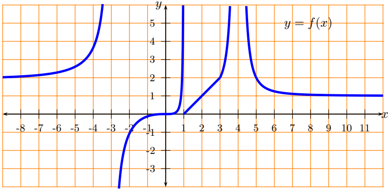
35. The function $f(x) = \dfrac{x}{1+\ln x}$ is discontinuous at

- [ ] -1
- [ ] 0
- [ ] e^{-1}
- [ ] e

36. The value of a that makes the given function continuous on $\mathbb{R}$ $f(x) = \begin{cases} 5x-2, & x \geq 2; \\ ax^2+4, & x < 2. \end{cases}$ is

- [ ] -1
- [ ] 2
- [ ] 0
- [ ] 1

37. The accompanying figure shows the graph of $y = f(x)$. Then the horizontal asymptotes for f are



- [ ] $y = -3,\ y = 1,\ y = 4$
- [ ] $x = 2,\ x = 1$
- [ ] $y = -3,\ y = 4,\ y = 1$
- [ ] $y = 2,\ y = 1$

38. If $y = (x - \sqrt{x})(x + \sqrt{x})$, then $y' =$

- [ ] $(1 - \frac{1}{2\sqrt{x}})(1 + \frac{1}{2\sqrt{x}})$
- [ ] $x - \frac{1}{2\sqrt{x}}$
- [ ] $2x - 1$
- [ ] $x + \frac{1}{2\sqrt{x}}$

39. If $f(x) = (3x - 2x^2)(5 + 4x)$, then $f'(x) =$

- ☐ $12 - 16x$
- ☐ $24x^2 - 4x - 15$
- ☐ $-24x^2 + 4x + 15$
- ☐ $16x - 12$

40. If $y = xe^x + x$, then $y'' =$

- ☐ $(x + 1)e^x$
- ☐ $(x + 2)e^x$
- ☐ xe^x
- ☐ e^x

41. $\lim_{x \to -\infty} \dfrac{3x - \sin x}{\cos x + x} =$

- ☐ DNE
- ☐ -3
- ☐ 3
- ☐ 1

42. $\lim_{x \to -\infty} \dfrac{e^x - 2e^{-x}}{e^x + e^{-x}} =$

- ☐ 1
- ☐ 0
- ☐ -2
- ☐ DNE

43. $\lim_{x \to \infty} \dfrac{e^x - 2e^{-x}}{e^x + e^{-x}} =$

- ☐ -2
- ☐ 1
- ☐ DNE
- ☐ 0

44. If $f(x) = e^2$ Then $f'(x) =$

- ☐ $2e$
- ☐ 0
- ☐ e
- ☐ e^2

45. The point on the curve $f(x) = x + e^x$ for which the tangent line is parallel to $y = 3x + 1$ is

- ☐ $(1, 1 + e)$
- ☐ $(\ln 2, 2 + \ln 2)$
- ☐ $(2, 2 + e^2)$
- ☐ $(0, 1)$

End

Score: Correct

Answers:

Points: Percent:

Letter Grade:

Solutions to Quizzes

Solution to 1. The function is the quotient of two functions x and $1-e^x$ with domains $\mathbb{R}$. Hence the domain is $\mathbb{R}$ except the zeroes of the bottom. $1-e^x = 0 \Rightarrow e^0 = 1 = e^x \Rightarrow x = 0$. Hence $D(f) = D(x) \cap D(1-e^x) \setminus \{x : 1-e^x = 0\} = \{\mathbb{R} \cap \mathbb{R}\} = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$. ■

Solution to 2. The function f is defined if and only if $2^x - 32 \geq 0 \Rightarrow 2^x \geq 2^5 \Rightarrow x \geq 5$. Hence $D(f) = [5, \infty)$. ■

Solution to 3.

$$\lim_{t\to 0}\tan\left(4\pi-\frac{\pi\sin t}{t}\right)=\tan\left(\lim_{t\to 0}(4\pi-\frac{\pi\sin t}{t})\right)\quad \tan x \text{ is continuous.}$$

$$=\tan\left(4\pi-\pi\lim_{t\to 0}\frac{\sin t}{t}\right)$$

$$=\tan\left(4\pi-\pi\right)$$

$$=\tan\left(3\pi\right)=0.$$

■

Solution to 4. Since $\lim_{x\to\infty}(1-3x^2)=-\infty$, $\lim_{x\to\infty}(4x^2+x+1)=\infty$ we have I.F. type $-\infty/\infty$. Divide each term in the numerator and each term in the denominator by the highest power in the denominator.

$$\lim_{x\to\infty}\frac{1-3x^2}{4x^2+x+1}=\lim_{x\to\infty}\frac{\dfrac{1-3x^2}{x^2}}{\dfrac{4x^2+x+1}{x^2}}$$

$$=\lim_{x\to\infty}\frac{\dfrac{1}{x^2}-\dfrac{3x^2}{x^2}}{\dfrac{4x^2}{x^2}+\dfrac{x}{x^2}+\dfrac{1}{x^2}}$$

$$=\lim_{x\to\infty}\frac{\dfrac{1}{x^2}-3}{4+\dfrac{1}{x}+\dfrac{1}{x^2}}$$

$$=\frac{0-3}{4+0+0}=\frac{-3}{4}$$

■

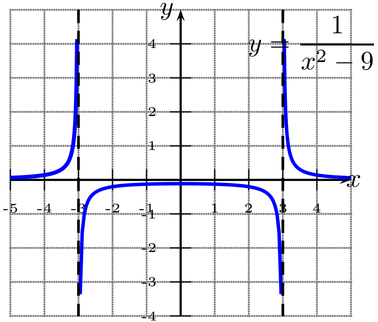
Solution to 5.

$$\lim_{x\to -3^+} \frac{1}{x^2-9} = \lim_{x\to -3^+} \frac{1}{(x-3)(x+3)}$$ A direct substation gives I.F. 1/0. If $x > -3$

$$= \lim_{x\to -3^+} \frac{1}{(x-3)(x+3)}$$ and near -3,then $1 > 0$, $x-3<0$, $x+3>0$.

$$= \lim_{x\to 3^+} \frac{\overset{+}{1}}{(x\overset{-}{-}3)(x\overset{+}{+}2)}$$

$$= -\infty$$



Solution to 6.

$$\lim_{x\to4}\frac{x-4}{x^2-2x-8} = \lim_{x\to2}\frac{x-4}{x^2-2x-8}$$ A direct substation will give us I.F. 0/0

$$= \lim_{x\to4}\frac{(x-4)}{(x-4)(x+2)}$$ Factoring $x-4$ from denominator

$$= \lim_{x\to4}\frac{\cancel{(x-4)}}{\cancel{(x-4)}(x+2)}$$

$$= \lim_{x\to4}\frac{1}{x+2} = \frac{1}{4+2} = \frac{1}{6}.$$

■

Solution to 7. A direct substation will give us I.F. 0/0. Now,

$$\lim_{x\to 1}\frac{x-1}{\sqrt{x^2+3}-2}=\lim_{x\to 1}\frac{x-1}{\sqrt{x^2+3}-2}\cdot\frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}\quad\text{Multiply by } 1=\frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}$$

$$=\lim_{x\to 1}\frac{(x-1)(\sqrt{x^2+3}+2)}{(\sqrt{x^2+3})^2-(2)^2}$$

$$=\lim_{x\to 1}\frac{(x-1)(\sqrt{x^2+3}+2)}{x^2-1}\quad\text{Factoring } x-1 \text{ from bottom}$$

$$=\lim_{x\to 1}\frac{\cancel{(x-1)}(\sqrt{x^2+3}+2)}{\cancel{(x-1)}(x+1)}$$

$$=\lim_{x\to 1}\frac{\sqrt{x^2+3}+2}{x+1}$$

$$=\frac{\sqrt{1+3}+2}{1+1}$$

$$=\frac{4}{2}=2.$$

■

Solution to 8. since $\lim_{x\to 0} f(x) = 1 = f(0)$, then $f(x)$ is continuous at $x = 0$. ■

Solution to 9. From the graph we can see that the tangent line to the graph of $y = f(x)$ at -2 is horizontal $(y = 3)$ and hence its slope is zero. Now, since the first derivative is the slope of the tangent line to the graph at -2 then $f'(-2) = 0$. ■

Solution to 10. f is not differentiable at $x = 1$ since the derivative from the left of 1 approaches $-\infty$ and from the right of 1 approaches ∞. ■

Solution to 11.

$$y = \sqrt{2} \quad \text{the derivative of a constant is zero}$$
$$y' = 0.$$

■

Solution to 12. A direct substation gives I.F. 0/0. Note that if $x > -1$ or $x < -1$ near(close) to -1 then $x - 1 < 0 \Rightarrow |x-1| = -(x-1)$.

$$\begin{aligned}
\lim_{x\to -1} \frac{|x-1|-2}{x+1} &= \lim_{x\to 0} \frac{-(x-1)-2}{x+1} \\
&= \lim_{x\to 1} \frac{-x+1-2}{x+1} \\
&= \lim_{x\to 1} \frac{-x-1}{x+1} \\
&= \lim_{x\to 1} \frac{-(x+1)}{x+1} \\
&= \lim_{x\to 1} (-1) \\
&= -1
\end{aligned}$$

■

Solution to 13.

$$y' = \frac{\overbrace{(x-2)}^{\text{Bottom}}\ \overbrace{(3x+2)'}^{\text{Derivative of Top}} - \overbrace{(3x+2)}^{\text{Top}}\ \overbrace{(x-2)'}^{\text{Derivative of Bottom}}}{\underbrace{(x-2)^2}_{}^{\text{Bottom}^2}}$$

$$= \frac{(x-2)(3)-(3x+2)(1)}{(x-2)^2}$$

$$= \frac{3x-6-(3x+2)}{(x-2)^2}$$

$$= \frac{3x-6-3x-2)}{(x-2)^2}$$

$$= \frac{-8}{(x-2)^2}.$$

■

Solution to 14. Notice that $f(x) = \sqrt{2-|x|}$ is an even root function, then it is continuous on its domain which is the set of real number such that $2-|x| \geq 0$. Now

$$\begin{aligned} 2-|x| \geq 0 &\Leftrightarrow -|x| \geq -2 \\ &\Leftrightarrow |x| \leq 2 \quad \Leftrightarrow -2 \leq x \leq 2. \end{aligned}$$

Hence f is continuous on $[-2, 2]$.

■

Solution to 15. The slope of the tangent line to $f(x) = x^2 + 2x$ at $x = 1$ is $f'(1)$.

$$f(x) = x^2 + 2x \Rightarrow f'(x) = 2x + 2.$$

Hence

$$\text{the slope of the tangent } = f'(1) = 2(1) + 2 = 4.$$

Also $f(1) = (1)^2 + 2(1) = 3$. Now, we have $m = 4$ and $(1, 3)$, hence

$$y - y_1 = m(x - x_1) \Rightarrow y - 3 = 4(x - 1) \Rightarrow y - 3 = 4x - 4 \Rightarrow y = 4x - 1.$$

■

Solution to 16.

$$
\begin{aligned}
F(x) &= x^3 - 27x \\
F'(x) &= 3x^2 - 27 = 3(x^2 - 9) \\
F'(x) &= 0 \\
3(x^2 - 9) &= 0 \\
x &= \pm 3.
\end{aligned}
$$

■

Solution to 17. Since $\lim_{x\to 3} \frac{x^2-2x-3}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+1)}{x-3} = \lim_{x\to 3}(x+1) = 4$, and $\lim_{x\to 3} \sqrt{3x+7} = 4$, then by The Squeeze Theorem we have $\lim_{x\to 3} f(x) = 4$. ■

Solution to 18. Write $f(x) = \dfrac{(x+3)(x-1)}{x(x-3)(x+3)}$. The zeroes of the denominator are -3, 0, and 3. To check that $x=-3$ is a vertical asymptote or not we take the limit at -3 from both sides. $\lim\limits_{x\to -3} f(x) = \lim\limits_{x\to -3} \dfrac{\cancel{(x+3)}(x-1)}{x\cancel{(x+3)}(x-3)} = \lim\limits_{x\to -3} \dfrac{x-1}{x(x-3)} = \dfrac{-4}{18} = \dfrac{-2}{9}$. Hence $x=-3$ is not a vertical asymptote. To check that $x=3$ we take the limit $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} \dfrac{\overset{+}{(x+3)(x-1)}}{\underset{}{\overset{+}{x(x-3)(x+3)}}} = \infty$. Hence $x=3$ is a vertical asymptote. To check that $x=0$ we take the limit $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \dfrac{\overset{-}{(x+3)(x-1)}}{\overset{-}{x(x-3)(x+3)}} = \infty$. Hence $x=0$ is a vertical asymptote. Thus the function has vertical asymptote at $x=0$, and $x=3$. ■

Solution to 19.

$$\begin{aligned} y &= \frac{x^3 + x^2 - x}{x^2} \\ y &= x + 1 - x^{-1} \\ y' &= 1 + x^{-2} \\ y' &= 1 + \frac{1}{x^2}. \end{aligned}$$

■

Solution to 20. Looking at the graph we can see that as $x \to \infty$, we see that $f(x) \to 1$. Hence $\lim_{x \to \infty} f(x) = 1$ ■

Solution to 21.

$$\lim_{x\to-3}\frac{x^3+27}{x^2-9} = \lim_{x\to-3}\frac{x^3+27}{x^2-9}$$ A direct substitution will give us I.F. 0/0

$$= \lim_{x\to-3}\frac{(x+3)(x^2-3x+9)}{(x+3)(x-3)}$$ Factoring $x+3$ from denominator and numerator

$$= \lim_{x\to-3}\frac{\cancel{(x+3)}(x^2-3x+9)}{\cancel{(x+3)}(x-3)}$$

$$= \lim_{x\to-3}\frac{x^2-3x+9}{x-3}$$

$$= \frac{(-3)^2-3(-3)+9}{-3-3}$$

$$= \frac{27}{-6} = -\frac{9}{2}.$$

■

Solution to 22. Taking the limits as $x \to \pm\infty$, we get the following:

$$\lim_{x\to-\infty} \frac{3x-1}{\sqrt{x^2+x+1}} = \lim_{x\to-\infty} \frac{\frac{3x-1}{x}}{\frac{\sqrt{x^2+x+1}}{x}} = \lim_{x\to-\infty} \frac{\frac{3x-1}{x}}{\frac{\sqrt{x^2+x+1}}{-\sqrt{x^2}}}$$

$$= \lim_{x\to-\infty} \frac{3-\frac{1}{x}}{-\sqrt{\frac{x^2+x+1}{x^2}}} = \lim_{x\to-\infty} \frac{3+\frac{1}{x}}{-\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}} = -3.$$

Similarly, we have

$$\lim_{x\to\infty} \frac{3x-1}{\sqrt{x^2+x+1}} = \lim_{x\to\infty} \frac{\frac{3x-1}{x}}{\frac{\sqrt{x^2+x+1}}{x}} = \lim_{x\to\infty} \frac{\frac{3x-1}{x}}{\frac{\sqrt{x^2+x+1}}{\sqrt{x^2}}}$$

$$= \lim_{x\to\infty} \frac{3-\frac{1}{x}}{\sqrt{\frac{x^2+x+1}{x^2}}} = \lim_{x\to\infty} \frac{3+\frac{1}{x}}{\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}} = \frac{3+0}{\sqrt{1+0}} = 3.$$

Therefore $y = \pm 3$ are horizontal asymptotes. ■

Solution to 23. A direct substation will give us I.F. $\infty - \infty$

$$\lim_{x\to-\infty}(x+\sqrt{x^2+x+1}) = \lim_{x\to-\infty}(x+\sqrt{x^2+x+1})\cdot\frac{x-\sqrt{x^2+x+1}}{x-\sqrt{x^2+x+1}}$$ use $(a-b)(a+b)=a^2-b^2$.

$$= \lim_{x\to-\infty}\frac{x^2-(x^2+x+1)}{x-\sqrt{x^2+x+1}}$$ use $x^2-(x^2+x+1)=x^2-x^2-x-$

$$= \lim_{x\to-\infty}\frac{-x-1}{x-\sqrt{x^2+x+1}}$$ divide up and down by x.

$$= \lim_{x\to-\infty}\frac{\frac{-x-1}{x}}{\frac{x-\sqrt{x^2+x+1}}{x}}$$ $x\to-\infty \Rightarrow x=-\sqrt{x^2}$.

$$= \lim_{x\to-\infty}\frac{-1-\frac{1}{x}}{1-\frac{\sqrt{x^2+x+1}}{-\sqrt{x^2}}}$$

$$= \lim_{x\to-\infty}\frac{-1-\frac{1}{x}}{1+\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}}$$

$$= \frac{-1-0}{1+\sqrt{1+0+0}} = \frac{-1}{2}$$

■

Solution to 24. From the graph of $f(x) = \sqrt{x+1}$ we see it is one-to-one and hence has an inverse. To find f^{-1}.

$$
\begin{aligned}
y &= \sqrt{x+1}, \quad y \geq 0 && \text{Let } y = f(x). \\
y^2 &= x+1, \quad y \geq 0 && \text{Square both sides.} \\
y^2 - 1 &= x, \quad y \geq 0 && \text{Solve for } x. \\
x^2 - 1 &= y, \quad x \geq 0 && \text{Interchange(Swap) } x \text{ and } y. \\
f^{-1}(x) &= x^2 - 1, \quad x \geq 0 && \text{Replace } y \text{ by } f^{-1}(x).
\end{aligned}
$$

Hence the inverse function of $f(x) = \sqrt{x+1}$ is $f^{-1}(x) = x^2 - 1, \quad x \geq 0$. ■

Solution to 25. From the graph of $f(x) = \dfrac{e^x}{1+3e^x}$ we see it is one-to-one and hence has an inverse. To find f^{-1}.

$$
\begin{aligned}
y &= \frac{e^x}{1+3e^x} && \text{Let } y = f(x). \\
y(1+3e^x) &= e^x && \text{if } a = \frac{b}{c} \Rightarrow ac = b. \\
y + 3ye^x &= e^x && \text{combine the like terms} \\
y &= e^x - 3ye^x && \text{take } e^x \text{ as common factor} \\
y &= e^x(1-3y) && \text{divide by } 1-3y \\
\frac{y}{1-3y} &= e^x && \text{Solve for } x. \\
\ln\left(\frac{y}{1-3y}\right) &= x && \text{Interchange(Swap) } x \text{ and } y. \\
\ln\left(\frac{x}{1-3x}\right) &= y
\end{aligned}
$$

Hence the inverse function of $f(x) = \dfrac{e^x}{1+3e^x}$ is $f^{-1}(x) =$

$\ln\left(\dfrac{x}{1-3x}\right)$. ■

Solution to 26.

$$\begin{aligned} \ln(\ln x) &= 1 && \text{Take } e \text{ for both sides} \\ \ln x &= e^1 && e^{\ln(\ln x)} = \ln x. \\ x &= e^e && \text{Take } e \text{ for both sides and note } e^{\ln x} = x \end{aligned}$$

■

Solution to 27. For $f(x) = \ln(3-x)$ to be defined we must have $3 - x > 0 \Rightarrow -x > -3$ and hence $x < 3$. Therefore $D(\ln(3-x)) = (-\infty, 3)$. ■

Solution to 28. For $f(x) = \ln(2 + \ln x)$ to be defined we must have $2 + \ln x > 0 \Rightarrow \ln x > -2$ and hence $x > e^{-2}$. Therefore $D(\ln(2 + \ln x)) = (e^{-2}, \infty)$. ■

Solution to 29. Since $[\![x]\!]$ is continuous at e then $\lim_{x \to e^+} [\![x]\!] = [\![e]\!] = 2.$ ■

Solution to 30. Since f is defined by different expressions for $x < 3$ and $x > 3$, we have to find $\lim_{x\to 3^+} f(x)$, and $\lim_{x\to 3^-} f(x)$.

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{x^2+21x-72}{x-3}$$ $x \to 3^+ \Rightarrow x > 3$. A direct substation will give

$$= \lim_{x\to 3^+} \frac{(x-3)(x+24)}{x-3}$$ Factoring $x-3$ from top and bottom

$$= \lim_{x\to 3^+} \frac{\cancel{(x-3)}(x+24)}{\cancel{x-3}} = 27$$

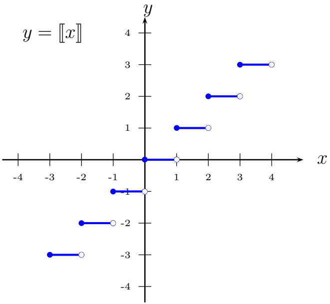
$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^3-27}{x-3}$$ $x \to 3^- \Rightarrow x < 3$. A direct substation will give

$$= \lim_{x\to 3^-} \frac{(x-3)(x^2+3x+9)}{x-3}$$ Factoring $x-3$ from top and bottom

$$= \lim_{x\to 3^-} \frac{\cancel{(x-3)}(x^2+3x+9)}{\cancel{x-3}} = 27.$$

Hence $\lim_{x\to 3} f(x) = 27$ ■

Solution to 31. Since $x \notin \mathbb{Z}$, then $[\![x]\!]$ is continuous and constant, hence $\dfrac{d}{dx}([\![x]\!]) = 0$.



Solution to 32. Since $\frac{5\pi}{2} \notin [\frac{-\pi}{2}, \frac{\pi}{2}]$, then $\sin^{-1}(\sin(\frac{5\pi}{2})) \neq \frac{5\pi}{2}$.

$$\begin{aligned}\sin^{-1}(\sin(\frac{5\pi}{2})) &= \sin^{-1}(1) \quad && \text{since} \sin(\frac{5\pi}{2}) = 1 \\ &= \frac{\pi}{2} && \text{since } \sin^{-1}(1) = \frac{\pi}{2}\end{aligned}$$

■

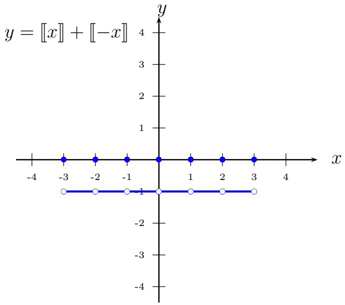
Solution to 33. Note that $\lim_{x\to 3^+} [\![x]\!] = 3$ and $\lim_{x\to 3^-} [\![x]\!] = 2$. Also $\lim_{x\to 3^+} [\![-x]\!] = -4$ and $\lim_{x\to 3^-} [\![-x]\!] = -3$.

$$\begin{aligned}\lim_{x\to 3^+} ([\![x]\!] + [\![-x]\!]) &= \lim_{x\to 3^+} [\![x]\!] + \lim_{x\to 3^+} [\![-x]\!] = 3 + (-4) \\ &= -1.\end{aligned}$$

and

$$\begin{aligned}\lim_{x\to 3^-} ([\![x]\!] + [\![-x]\!]) &= \lim_{x\to 3^-} [\![x]\!] + \lim_{x\to 3^-} [\![-x]\!] = 2 + (-3) \\ &= -1.\end{aligned}$$

Hence $\lim_{x\to 3} ([\![x]\!] + [\![-x]\!]) = -1$



$\blacksquare$

Solution to 34. From the graph we see that $\lim_{x\to 4} f(x) = \infty$, hence $x = 4$ is vertical asymptote. From the graph we see that $\lim_{x\to 1^-} f(x) = \infty$, hence $x = 1$ is vertical asymptote. From the graph we see that $\lim_{x\to -3^+} f(x) = -\infty$, hence $x = -3$ is vertical asymptote. Hence f have vertical asymptotes at $x = -3,\ x = 1,\ x = 4$. ■

Solution to 35. Since $1 + \ln x = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1}$, then f is discontinuous at e^{-1}. ■

Solution to 36. f is continuous on the set of real numbers if it is continuous at $x = 2$ and f is continuous at $x = 2$ if and only if $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$. Now

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (5x - 2) = 5(2) - 2 = 8$ and

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (ax^2 + 4) = a(2)^2 + 4 = 4a + 4.$

Hence

$$4a + 4 = 8 \Leftrightarrow 4a = 4 \Leftrightarrow a = \frac{4}{4} = 1.$$

■

Solution to 37. From the graph we see that $\lim_{x\to\infty} f(x) = 1$, hence $y = 1$ is horizontal asymptote. From the graph we see that $\lim_{x\to-\infty} f(x) = 2$, hence $y = 2$ is horizontal asymptote. Hence f have horizontal asymptotes at $y = 2,\ y = 1$. ■

Solution to 38.

$$y = (x - \sqrt{x})(x + \sqrt{x}) \quad \text{simplify}$$
$$y = x^2 - x \quad \text{use } (a-b)(a+b) = a^2 - b^2$$
$$y' = 2x - 1$$

Solution to 39.

$$f'(x) = \underbrace{(3x-2x^2)}_{\text{First}} \cdot \underbrace{\frac{d}{dx}(5+4x)}_{\text{Derivative of second}} + \underbrace{(5+4x)}_{\text{Second}} \cdot \underbrace{\frac{d}{dx}(3x-2x^2)}_{\text{Derivative of first}}$$

$$= (3x-2x^2)(4) + (5+4x)(3-4x)$$

$$= (12x-8x^2) + (15+12x-20x-16x^2)$$

$$= -24x^2+4x+15.$$

■

Solution to 40.

$$
\begin{aligned}
y &= xe^x + x \\
y' &= xe^x + e^x + 1 \\
y'' &= xe^x + e^x + e^x = (x+2)e^x.
\end{aligned}
$$

■

Solution to 41. Since $\lim_{x\to-\infty}(3x-\sin x)=-\infty$ and $\lim_{x\to-\infty}(\cos x+x)=-\infty$, we have I.F. (∞/∞)

$$\lim_{x\to-\infty}\frac{3x-\sin x}{\cos x+x}=\lim_{x\to-\infty}\frac{\frac{3x-\sin x}{x}}{\frac{\cos x+x}{x}} \qquad \text{divide by } x$$

$$=\lim_{x\to-\infty}\frac{3-\frac{\sin x}{x}}{\frac{\cos x}{x}+1} \qquad \lim_{x\to-\infty}\frac{\sin x}{x}=0=\lim_{x\to-\infty}\frac{\cos x}{x}$$

$$=\frac{3-0}{0+1}=3.$$

■

Solution to 42. Since $\lim_{x\to-\infty}(e^x - 2e^{-x}) = -\infty$ and $\lim_{x\to-\infty}(e^x + e^{-x}) = \infty$, we have I.F. (∞/∞)

$$\lim_{x\to-\infty}\frac{e^x - 2e^{-x}}{e^x + e^{-x}} = \lim_{x\to-\infty}\frac{\dfrac{e^x - 2e^{-x}}{e^{-x}}}{\dfrac{e^x + e^{-x}}{e^{-x}}} \qquad \text{divide by } e^{-x}$$

$$= \lim_{x\to-\infty}\frac{e^{2x} - 2}{e^{2x} + 1} \qquad \lim_{x\to-\infty} e^{2x} = 0$$

$$= \frac{0-2}{0+1} = -2.$$

■

Solution to 43. Since $\lim_{x\to\infty}(e^x - 2e^{-x}) = \infty$ and $\lim_{x\to\infty}(e^x + e^{-x}) = \infty$, we have I.F. (∞/∞)

$$\lim_{x\to\infty}\frac{e^x - 2e^{-x}}{e^x + e^{-x}} = \lim_{x\to\infty}\frac{\dfrac{e^x - 2e^{-x}}{e^x}}{\dfrac{e^x + e^{-x}}{e^x}} \quad \text{divide by } e^x$$

$$= \lim_{x\to\infty}\frac{1 - 2e^{-2x}}{1 + e^{-2x}} \quad \lim_{x\to\infty} e^{-2x} = 0$$

$$= \frac{1-0}{1+0} = 1.$$

Solution to 44. The function $f(x) = e^2$ is a constant, hence $f'(x) = 0$ ■

Solution to 45. The slope of the given line is 3. We want to find the values of x for which is $f'(x) = 3$. Now, $f'(x) = 1 + e^x$. $f'(x) = 3 \Rightarrow 1 + e^x = 3 \Rightarrow e^x = 2 \Rightarrow x = \ln 2$. $f(\ln 2) = 2 + \ln 2$. Hence the point is $((\ln 2, 2 + \ln 2)$. ■